An application of the Fourier method in acoustics

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Abstract. We consider the behaviour of sound in the bounded non-convex room. This configuration appears in the engines of spacecrafts. The wave equation is briefly reintroduced. On a part of the boundary nonhomogeneous Dirichlet conditions are given, on the rest of the boundary homogeneous Neumann conditions. The classical and weak solution are defined for this initial and boundary value problem. The uniqueness of solution is observed. The Fourier method is applied in constructing a solution candidate, and sufficient conditions are given which provide that this candidate is the weak or classical solution of the problem. Finally a method for determining the proper frequencies is proposed.

1. Introduction. Let us consider a box with a hole $U$ in a closed room (Fig. 1). On the inner wall of the box there is a region $S$ which operates as the sound source. On $S$ the air pressure is known. We are interested in the behaviour of sound in this configuration, and the empirical method for determining the proper frequencies.

In Chapters 2 and 3 we recall the basic notions describing sound waves. In Chapter 4 is our notation and the problem is formulated mathematically as a boundary value problem with Dirichlet and Neumann boundary conditions. The Fourier method is applied in Chapter 5 in construction of the solution. In Chapter 6 is a remark on the resonance phenomenon.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Figure 1.}
\end{figure}

2. The sound. Let us remind the basic concepts describing sound waves. For the more complete exposition the reader is referred to [3], especially its Chapter VIII.

The fluid at point $x$ and time $t$ is described by its pressure $p$, density $\rho$, velocity $v$, temperature $T$ and density of entropy $s$. These are connected to

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each other by the equations of fluid mechanics and thermodynamics. The sound is constituted by small vibrations of the compressible fluid, i.e.

\[(2.1) \quad \rho = \rho_0 + \rho_1, \quad p = p_0 + p_1,\]

where \(\rho_0\) and \(p_0\) are constants with respect to \(t\) and \(|\rho_1| \ll \rho_0, |p_1| \ll p_0\); \(v\) and \(\nabla v\) are small. The viscous effects are neglected and the processes are assumed to be adiabatic.

The continuity equation of mass and Euler’s equation for non-viscous fluid,

\[(2.2) \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \rho v = 0, \quad \frac{\partial v}{\partial t} + v \cdot \nabla v = -\frac{\nabla p}{\rho}\]

imply, as an approximation of the first order of smallness of \(\rho_1, p_1, v\) and \(\nabla v\),

\[(2.3) \quad \frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot v = 0\]

\[(2.4) \quad \frac{\partial v}{\partial t} + \frac{\nabla p_1}{\rho_0} = 0\]

By the assumption of adiabatic processes, Taylor’s expansion of \(p = p(s, \rho)\) at \((s, \rho_0)\) gives, as the first order approximation,

\[(2.5) \quad p_1 = \left(\frac{\partial p}{\partial \rho_0}\right)_s \rho_1.\]

Hence, it follows from (2.3) that

\[(2.6) \quad \frac{\partial p_1}{\partial t} + \rho_0 \left(\frac{\partial p}{\partial \rho_0}\right)_s \nabla \cdot v = 0.\]

The equations (2.4) and (2.6) contain the unknown functions \(p_1\) and \(v\) which describe completely the sound wave. Let us take the divergence of (2.4) and apply (2.6) and the smallness of \(\nabla \rho_0\). The result is the wave equation for \(p_1\),

\[(2.7) \quad \frac{\partial^2 p_1}{\partial t^2} - c^2 \Delta p_1 = 0\]

where

\[(2.8) \quad c = \left(\frac{\partial p}{\partial \rho_0}\right)_s^{\frac{1}{2}}\]

is the velocity of sound. It depends typically only on the temperature and thus we can assume it to be a constant.
3. The boundary conditions. If the fluid is restricted by a wall \( \Gamma_1 \) whose velocity is \( \mathbf{u} = \mathbf{u}(x, t) \) and normal vector \( \nu = \nu(x, t) \) then the normal component of velocity is continuous at the wall, i.e. \( \nu \cdot \nu = \mathbf{u} \cdot \mathbf{v} \). By (2.4) this implies

\[
\frac{\partial p_1}{\partial \nu} = \nu \cdot \nabla p_1 = -\rho_0 \frac{\partial}{\partial t} \nu \cdot \mathbf{u} + \rho_0 \mathbf{v} \cdot \frac{\partial}{\partial t} \nu \text{ on } \Gamma_1.
\]

If \( \nu \) does not depend on \( t \) this reduces to Neumann boundary conditions. If the pressure is given at boundary \( \Gamma_2 \) by the function \( f \) then we have the Dirichlet boundary conditions

\[
P_1 = f \text{ on } \Gamma_2.
\]

4. Our notation and problem. We shall use the Sobolev space \( H^1(\Omega) \), the space \( C_0^\infty(\Omega) \), the space \( L^2(0, T; V) \) of square integrable \( V \)-valued functions on the interval \( [0, T] \) and the Sobolev spaces \( H^1(0, T; V) \) and \( H^2(0, T; V) \) where \( V \) is a real Banach space. The dual space of \( V \) is denoted by \( V^* \) and the dual pairing between \( V \) and \( V^* \) by \( \langle \cdot, \cdot \rangle_V \); if \( y \in V \) and \( z \in V^* \) then \( \langle z, y \rangle_V = z(y) \). The inner product of the Hilbert space \( H \) is denoted by \( \langle \cdot, \cdot \rangle_H \) and the norm of the space \( X \) is denoted by \( \| \cdot \|_X \). For these concepts the reader may refer \([1]\) and \([5]\).

Let \( \Omega \subset \mathbb{R}^n, n = 1, 2, \ldots \) be an open bounded set whose boundary \( \partial \Omega = \Gamma \) is Lipschitzian. In our acoustical problem \( \Omega \) represents the interior of the box and the rest of the room. Let \( S \subset \Gamma \) be a non-void open set, \( f \in H^2(0, T) \) and \( c \in \mathbb{R}_+ \).

Our problem is the following boundary value problem

\[
\begin{align*}
\frac{\partial^2 p_1}{\partial t^2} - c^2 \Delta p_1 &= 0 \text{ in } \Omega \times \mathbb{R}_+, \\
\frac{\partial p_1}{\partial \nu} &= 0 \text{ on } \Gamma_1 \times \mathbb{R}_+ = (\Gamma \setminus S) \times \mathbb{R}_+, \\
p_1 &= f \text{ on } S \times \mathbb{R}_+.
\end{align*}
\]

Let \( g(t) = -f''(t) \) and \( y \) be the solution of

\[
\begin{align*}
y_{tt} - c^2 \Delta y &= g \text{ in } \Omega \times \mathbb{R}_+, \\
\frac{\partial y}{\partial \nu} &= 0 \text{ on } \Gamma_1 \times \mathbb{R}_+, \\
y &= 0 \text{ on } S \times \mathbb{R}_+.
\end{align*}
\]

Then \( y + f \) is a solution of (4.1).

Let us impose the initial conditions

\[
y(x, 0) = y_0(x), \quad y_t(x, 0) = y_1(x), \quad x \in \Omega.
\]
Consider the real Hilbert space
\[ V = \{ \phi \in H^1(\Omega) \mid \phi = 0 \text{ on } S \} \]
whose inner product and norm are given by
\[ (\phi, \psi)_V = \int_\Omega \nabla \phi \cdot \nabla \psi \, d^n x, \quad \|\phi\|_V = (\phi, \phi)_V^{\frac{1}{2}}. \]

By [7, Thm. 1.9, p. 20], the norm of \( V \) is equivalent (in \( V \)) to the norm of \( H^1(\Omega) \). Identifying \( L^2(\Omega) \) with its dual we have
\[ V \subset L^2(\Omega) \subset V^* \]
algebraically and topologically.

Let \( T > 0, y_0, y_1 \in V^* \) and \( g \in L^2(0, T, V^*) \). A function \( y \in H^2(0, T; V^*) \cap L^2(0, T, V) \) satisfying \( y(0) = y_0, y_1(0) = y_1 \) and
\[ (4.4) \quad (y''(t), \phi)_V + c^2(y(t), \phi)_V = (g(t), \phi)_V \quad \forall \phi \in V, \text{ a.e. } t \in [0, T] \]
is called weak solution of (4.2)-(4.3).

**Lemma 4.1.** The weak solution of (4.2)-(4.3) is unique.

**Proof:** Let \( y \) be the difference of two weak solutions of (4.2)-(4.3). Then \( y'(0) = y(0) = 0 \) and
\[ (y'(t), \phi)_V + c^2(\int_0^t y(s) \, ds, \phi)_V = 0 \quad \forall \, t \in [0, T], \, \phi \in V. \]

Thus, especially,
\[ (y'(t), y(t))_V + c^2(\int_0^t y(s) \, ds, y(t))_V = 0 \quad \forall \, t \in [0, T]. \]

The first term is by [1, p. 62],
\[ \frac{1}{2} \frac{d}{dt} (y(t), y(t))_V = \frac{1}{2} \frac{d}{dt} \|y(t)\|_{L^2(\Omega)}^2. \]

Thus, by integration over \([0, t]\) we get \( \|y(t)\|_{L^2(\Omega)} = 0 \), i.e. \( y(t) = 0 \), for each \( t \in [0, T] \).

The existence results for the solution of (4.2)-(4.3) can be found in [5, p. 297] and [4, pp. 178-183].
5. The Fourier method. Let us apply to our problem the Fourier method applied by [4, pp. 33-34] in the case of homogenous Dirichlet boundary conditions on the whole \( \partial \Omega \).

Let us consider the problem

\begin{align}
(5.1a) & \quad -\Delta u = f \text{ a.e. in } \Omega, \\
(5.1b) & \quad u = 0 \text{ on } S, \\
(5.1c) & \quad \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_1 = \Gamma \setminus S
\end{align}

where \( f \in L^2(\Omega) \). Its variational (i.e. weak ) formulation is

\begin{equation}
(5.2) \quad \text{Find } u \in V \text{ such that } \int_{\Omega} \nabla u \cdot \nabla \phi \, d^m x = \int_{\Omega} f \phi \, d^m x \quad \forall \, \phi \in V.
\end{equation}

According to Lax-Milgram Theorem [7, p. 38], we obtain that for each \( f \in L^2(\Omega) \) there is a unique solution \( u_f \in V \) for (5.2). The operator

\begin{equation}
(5.3) \quad P : L^2(\Omega) \mapsto V, \ P f = u_f
\end{equation}

is linear and continuous. Let us denote by \( Q \) the restriction of \( P \) to \( V \), \( Q = P|_V \).

**Lemma 5.1.** The operator \( Q : V \mapsto V \) is linear, continuous, compact and selfadjoint. Moreover, its kernel is \( \{0\} \).

It follows from the Hilbert-Schmidt theorem [2, p. 283] that \( Q \) has an enumerable set of eigenvalues \( \{\lambda_n\}_{n=1}^\infty \) and corresponding eigenvectors \( \{u_n\}_{n=1}^\infty \subset V \) which form a Hilbertian basis in \( V \). Moreover, \( \lambda_n > 0, \ n = 1, 2, \ldots \) and \( \lambda_n \to 0 \), as \( n \to \infty \).

So, we have

\[ Q u_n = \lambda_n u_n, \ n = 1, 2, \ldots \]

In other words, \( u_n \) are weak solutions for

\begin{align}
(5.4a) & \quad -\Delta u_n = \mu_n^2 u_n \text{ in } \Omega, \\
(5.4b) & \quad u_n = 0 \text{ on } S, \\
(5.4c) & \quad \frac{\partial u_n}{\partial n} = 0 \text{ on } \Gamma_1
\end{align}

with \( \mu_n^2 = \lambda_n^{-1} \).

**Lemma 5.2.** The set \( \{\mu_n u_n \mid n = 1, 2, \ldots \} \) is a Hilbertian basis of \( L^2(\Omega) \), and \( \{\mu_n^2 u_n \mid n = 1, 2, \ldots \} \) is a Hilbertian basis of \( V^* \).

Since \( 1 \in L^2(\Omega) \),

\begin{equation}
(5.5) \quad 1 = \sum_{n=1}^\infty a_n v_n \text{ in } L^2(\Omega)
\end{equation}


where
\[ a_n = \mu_n \int_{\Omega} u_n(x) \, dx, \quad v_n = \mu_n u_n, \quad n = 1, 2, \ldots \]

We shall try to find \( y \), the solution of (4.2)-(4.3), in the following form:

\[ y(x, t) = \sum_{n=1}^{\infty} b_n(t) v_n(x). \tag{5.6} \]

So, *formally*, we obtain the equations

\[
\begin{align*}
(5.7a) & \quad b_n''(t) + \mu_n^2 c_n^2 b_n(t) = a_n g(t), \quad t \in \mathbb{R}_+, n = 1, 2, \ldots \\
(5.7b) & \quad b_n(0) = \hat{b}_n, \quad b_n'(0) = \dot{b}_n, \quad n = 1, 2, \ldots
\end{align*}
\]

where \( \hat{b}_n \) and \( \dot{b}_n \) are the Fourier coefficients of \( y_0 \) and \( y_1 \):

\[ y_0(x) = \sum_{n=1}^{\infty} \hat{b}_n v_n(x), \quad y_1(x) = \sum_{n=1}^{\infty} \dot{b}_n v_n(x), \quad x \in \Omega. \]

Let us recall [2, p. 176] that

\[ y_0 \in V \iff \sum_{n=1}^{\infty} |\hat{b}_n| \mu_n^2 < \infty, \quad y_1 \in L^2(\Omega) \iff \sum_{n=1}^{\infty} |\dot{b}_n|^2 < \infty. \]

After solving (5.7) we can write:

\[ y(x, t) = \sum_{n=1}^{\infty} \left( \hat{b}_n \cos c_n \mu_n t + \frac{\dot{b}_n}{c_n \mu_n} \sin c_n \mu_n t \right) v_n(x) \]
\[ + \sum_{n=1}^{\infty} \frac{a_n}{c_n \mu_n} \int_{0}^{t} g(s) \sin c_n \mu_n (t-s) \, ds \, v_n(x). \tag{5.8} \]

**Theorem 5.1.** Let \( y_0 \in V \), \( y_1 \in L^2(\Omega) \) and \( g \in L^2(0, T) \). Then \( y \), given by (5.8), belongs to

\[ C([0, T]; V) \cap C^1([0, T]; L^2(\Omega)) \cap H^2(0, T; V^*), \]

and it is the unique weak solution of (4.2)-(4.3).

Let us denote

\[ W = \{ z \in H^1(\Omega) \mid \Delta z \in L^2(\Omega) \}, \quad ||z||_W^2 = ||z||_{H^1(\Omega)}^2 + ||\Delta z||_{L^2(\Omega)}^2. \]

The space \((W, ||\cdot||_W)\) turns out to be a Banach space.
THEOREM 5.2. Let \( y_0 \in W \cap V, y_1 \in V \) and \( g \) be absolutely continuous on \([0, T]\) such that \( g' \in L^1(0, T) \). Then \( y \), given by (5.8), belongs to
\[
C([0, T]; V \cap W) \cap C^1([0, T]; V) \cap C^2([0, T]; L^2(\Omega)),
\]
and it is the unique classical solution of (4.2)-(4.3).

THE PROOF OF LEMMA 5.1: The linearity, continuity and the nondegeneracy are obvious. Let \((f_n)\) be a bounded sequence of \( V \subset H^1(\Omega) \). By Rellich's theorem [7, p. 17] there is a subsequence \( f_{n_j} \) and \( f \in L^2(\Omega) \) such that \( f_{n_j} \to f \). Hence
\[
\|Qf_{n_j} - Qf\|_V = \sup_{\|\phi\|_V \leq 1} (Q(f_{n_j} - f), \phi)_V
= \sup_{\|\phi\|_V \leq 1} \int_{\Omega} (f_{n_j} - f)\phi \, dx \leq \text{Const.} \|f_{n_j} - f\|_{L^2(\Omega)} \to 0
\]
as \( j \to \infty \), i.e. \( Q \) is compact. Let \( f, g \in V \). Then
\[
(Qf, g)_V = \int_{\Omega} \nabla u_f \cdot \nabla g \, dx = \int_{\Omega} fg \, dx = \int_{\Omega} \nabla f \cdot \nabla u_g \, dx = (f, Qg)_V,
\]
i.e. \( Q \) is selfadjoint.

THE PROOF OF LEMMA 5.2: Let us prove the first statement. The orthonormality is obvious. For the maximality it suffices to show that \( (g, \mu_i u_i)_{L^2(\Omega)} = 0 \) \( \forall i = 1, 2, \ldots \) implies \( g = 0 \), cf. [2, Thm. 4, p. 179].

Let \( g \in L^2(\Omega) \) be such that \( (g, \mu_i u_i)_{L^2(\Omega)} = 0 \), for each \( i = 1, 2, \ldots \) Then by (5.2), \( (u_g, u_i)_V = 0, \forall i = 1, 2, \ldots \) This implies \( u_g = 0 \) because \({u_n}\) is a Hilbertian basis of \( V \). Hence \( g = 0 \) in \( L^2(\Omega) \).

The statement on a basis of \( V^* \) is proved by the same idea. The orthonormality follows from \((u_n, u_m)_{V^*} = \frac{1}{2}(\|u_n + u_m\|_{V^*}^2 - \|u_n\|_{V^*}^2 - \|u_m\|_{V^*}^2)\). For example, \( \|u_n\|_{V^*} = \sup_{\|\phi\|_{V^*} \leq 1} (u_n, \phi)_{L^2(\Omega)} = \mu_n^{-2} \). In proving the maximality we first notice that \( (g, \mu^2 u_n)_{V^*} = 0 \) \( \forall n = 1, 2, \ldots \) implies that \( (g, y)_{V^*} = 0 \) \( \forall y \in L^2(\Omega) \).

Since \( L^2(\Omega) \) is dense in \( V^* \), there exists, for each \( \varepsilon > 0 \), \( g \in L^2(\Omega) \) such that
\[
\epsilon > \|g - g_\varepsilon\|_{V^*}^2 = \|g\|_{V^*}^2 - 2(g, g_\varepsilon)_{V^*} + \|g_\varepsilon\|_{V^*}^2 \geq \|g\|_{V^*}^2.
\]
Thus \( g = 0 \) and \( \{\mu_n^2 u_n \mid n = 1, 2, \ldots \} \) is a Hilbertian basis of \( V^* \).

THE PROOF OF THEOREM 5.1: Since
\[
|b_n(t)\mu_n|^2 \leq 2(\mu_n \tilde{b}_n)^2 + \frac{2}{c^2}(b_n)^2 + 2a_n^2 M_T,
\]
the series of \( \sum_{n=1}^{\infty} (b_n(t)\mu_n)^2 \) converges uniformly on \([0, T]\). So noticing that
\[
\| \sum_{n=m}^{p} b_n(t)\mu_n u_n \|_V^2 = \sum_{n=m}^{p} |b_n(t)\mu_n|^2,
\]
we can get that the series of (5.8) converges in $V$ uniformly with respect to $t \in [0, T]$ and $y \in C([0, T]; V)$.

Now, consider the series

\[(5.9) \quad \tilde{y}(t) = \sum_{n=1}^{\infty} b_n'(t) \mu_n u_n, \quad \hat{y}(t) = \sum_{n=1}^{\infty} b_n''(t) \mu_n u_n.\]

We can show as above that the first series of (5.9) is uniformly convergent in $L^2(\Omega)$ with respect to $t \in [0, T]$ and $\tilde{y} \in C([0, T]; L^2(\Omega))$. Remark also that

\[\sum_{n=1}^{\infty} \mu_n^{-2} |b_n''(t)|^2 \leq \text{Const.} (1 + |g(t)|^2)\]

and

\[|| \sum_{n=m}^{p} b_n''(t) \mu_n u_n ||_V^2 = \sum_{n=m}^{p} \mu_n^{-2} |b_n''(t)|^2.\]

Hence the second series of (5.9) converges in $L^2(0, T; V^*)$ and $\hat{y} \in L^2(0, T; V^*)$. Now, let $\phi \in C_0^{\infty}(0, T)$. We have

\[\int_0^T \phi(t) \{ \sum_{n=1}^{m} b_n'(t) \mu_n u_n \} \, dt = - \int_0^T \phi'(t) \{ \sum_{n=1}^{m} b_n(t) \mu_n u_n \} \, dt.\]

Therefore, letting $m \to \infty$, we get

\[\int_0^T \phi(t) \tilde{y}(t) \, dt = - \int_0^T \phi'(t) y(t) \, dt\]

in $L^2(\Omega)$, which implies $\tilde{y} = y'$ in $\mathcal{D}'(0, T; L^2(\Omega))$, i.e. $y \in C^1([0, T]; L^2(\Omega))$. Similarly we can show that $y'' = \hat{y} \in L^2(0, T; V^*)$. Now, for each $n = 1, 2, \ldots$ we have

\[\langle y''(t), u_n \rangle_V + c^2 \langle y(t), u_n \rangle_V = \sum_{j=1}^{\infty} \langle b_j'(t) \mu_j u_j, u_n \rangle_V + c^2 \sum_{n=1}^{\infty} \langle b_j(t) \mu_j u_j, u_n \rangle_V\]

\[= b_n''(t) \mu_n^{-1} + b_n(t) c^2 \mu_n = \mu_n^{-1} a_n g(t),\]

for a.e. $t \in [0, T]$. Thus

\[\langle y''(t), \phi \rangle_V + c^2 \langle y(t), \phi \rangle_V = \langle g(t), \phi \rangle_{L^2(\Omega)} \quad \forall \phi \in V, \text{ a.e. } t \in [0, T],\]

i.e. $y$ is the weak solution of (4.2)-(4.3). Finally we observe that the initial conditions are satisfied by the construction of (5.8).

**The proof of Theorem 5.2:** It follows from $y_0 \in W \cap V$ and $y_1 \in V$ that $\sum_{n=1}^{\infty} \mu_n^4 b_n^2 < \infty$ and $\sum_{n=1}^{\infty} \mu_n^2 b_n^2 < \infty$. It turns out that $y'' \in C([0, T]; L^2(\Omega))$,
as in the proof of Theorem 5.1. Thus $y \in C^2([0, T]; L^2(\Omega))$. Similarly one can show that $y \in C^1([0, T]; V) \cap C([0, T]; V \cap W)$. The generalized partial derivatives $y_t, y_{tt}$ and $\Delta y$ are $y', y''$ and $-\sum_n b_n(t)\mu_n^3 u_n$, respectively; $D_i y$ exists, $i = 1, 2, \ldots, n$. All these are in the space $L^2(Q)$, $Q = [0, T] \times \Omega$. Thus the classical derivatives $y_{tt}$ and $\Delta y$ exist a.e. in $Q$. Hence the equation (4.2a) is satisfied a.e. in $Q$ by (4.4).

For the justification and the exact meaning of the boundary conditions we just refer to [5, pp. 297-298].

6. The resonance. If $g(t) = d \sin c \nu t$ where $\nu \approx \mu_n$, for some $n = 1, 2, \ldots$, then one obtains the resonance: the $n$-th term of (5.8) is very large, indeed the solution of (5.7) is

\begin{equation}
(6.1) \quad b_n(t) = \tilde{b}_n \cos c\mu_n t + \frac{1}{c\mu_n} \left( b_n - \frac{d\nu}{c\mu_n^2 - c\nu^2} \right) \sin c\mu_n t + \frac{d}{c^2 \mu_n^2 - c^2 \nu^2} \sin c\nu t.
\end{equation}

Thus

\begin{equation}
(6.2) \quad b_n(t) \approx \frac{2d}{c^2 (\mu_n^2 - \nu^2)} \cos \frac{c(\mu_n + \nu)t}{2} \sin \frac{c(\nu - \mu_n)t}{2}.
\end{equation}

This formula allows to develop the method to measure the frequencies of proper oscillations of sound in the room $\Omega$, by regulating the frequency of the sound in the source $S$.

Note that such acoustical phenomena as described above have been observed in motors of spaceships (type ARIANE) which use solid propergol (disposed in form of segments), [6].

REFERENCES


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