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AN INVERSE PROBLEM FOR THE TELEGRAPH SYSTEM

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An inverse problem arising from the nonlinear telegraph system (1)–(4) is formulated and investigated as an optimization problem. This leads us to study by D’Alembert formulæ whether the solution of (1)–(4) is continuous and differentiable with respect to the optimized parameter.

1 Introduction

Consider the following boundary value problem (BVP):

\[
\frac{\partial u}{\partial t} (t, x) + \frac{\partial v}{\partial x} (t, x) + Ru(t, x) \equiv f_1(t, x), \quad 0 < x < 1, \quad t > 0,
\]

\[
\frac{\partial v}{\partial t} (t, x) + \frac{\partial u}{\partial x} (t, x) + Gv(t, x) \equiv f_2(t, x), \quad 0 < x < 1, \quad t > 0,
\]

\[
ru(t, 0) + v(t, 0) \equiv 0, \quad u(t, 1) \in \beta v(t, 1), \quad t > 0,
\]

\[
u(0, x) = u_0(x), \quad v(0, x) = v_0(x), \quad 0 < x < 1,
\]

where \( r, \beta, R, G \subseteq \mathbb{R} \times \mathbb{R} \) are maximal monotone operators, \( u_0, v_0 : [0, 1] \rightarrow \mathbb{R} \), and \( f_1, f_2 : [0, \infty] \times [0, 1] \rightarrow \mathbb{R} \). This BVP models electrical circuits with a nonlinear resistance at \( x = 1 \) [3]. Moreover, this BVP is essentially similar to a boundary value problem, modelling the electromagnetic radiation of an antenna.

We are interested in an inverse (or identification) problem where one should find \( r \) when \( \beta, R, G, f_1, f_2, u_0, \) and \( v_0 \) are given and BVP has a
solution. However, as we shall see below, BVP has solution under quite general assumptions for any maximal monotone \( r \). Thus our identification problem is ill-posed, and hence we study the following optimization problem (P), where we are looking \( r \) such that the exact solution \( (u_r, v_r) \) of BVP is close enough to \( (\hat{u}, \hat{v}) \), the measured or desired solution of BVP.

Let \( T > 0, K \) be a set of some maximal monotone operators in \( \mathbb{R} \) and let \( X \) be a Banach space of some functions \([0, T] \times [0, 1] \mapsto \mathbb{R} \). The set \( K \) and the space \( X \) remain to be clarified later.

(P) Let \( (\hat{u}, \hat{v}) \in X \). Find \( r \in K \) such that \( r \) minimizes the function \( J \),

\[
J : K \mapsto \mathbb{R}, \quad J(r) = \frac{1}{2} \| (u_r, v_r) - (\hat{u}, \hat{v}) \|^2_X,
\]

where \((u_r, v_r)\) is the solution of BVP corresponding to \( r \). For the existence and the regularity of the solution for BVP see [2,1].

2 The case of unperturbed D’Alembertian

Let \( T > 0, R = G = 0 \) and \( f_1, f_2 : [0, T] \times [0, 1] \mapsto \mathbb{R} \). We extend \( f_1 \) and \( f_2 \) to be defined on \([0, T] \times \mathbb{R} \) by

\[
f_i(t, x) = f_i(t, 2 - x), \quad \text{if} \quad 1 < x \leq 2,
\]

\[
f_i(t, x) = f_i(t, -x), \quad \text{if} \quad -1 \leq x < 0,
\]

\( i = 1, 2, \) and so on. The general solution of (1)–(2) is given by the d’Alembert formulae (8)–(11); for each \( t \in [0, T] \) and \( x \in [0, 1] \),

\[
u(t, x) = \frac{1}{2} \left( \phi(x - t) - \psi(x + t) + h_1(t, x) - h_2(t, x) \right), \quad \text{(9)}
\]

where

\[
h_1(t, x) = \frac{1}{2} \int_{x-t}^{x} (f_1 + f_2)(t - x + s, s) \, ds, \quad \text{(10)}
\]

\[
h_2(t, x) = \frac{1}{2} \int_{x}^{x+t} (f_1 - f_2)(t + x - s, s) \, ds, \quad \text{(11)}
\]

and the functions \( \phi : [-T, 1] \mapsto \mathbb{R} \) and \( \psi : [0, 1 + T] \mapsto \mathbb{R} \) are determined by the initial and by the boundary conditions. Indeed, by (3),

\[
\phi(x) = u_0(x) + v_0(x), \quad \psi(x) = u_0(x) - v_0(x), \quad \text{if} \quad 0 \leq x \leq 1. \quad \text{(12)}
\]
Without loss of generality we may assume that \( T \leq 1 \). Then (4) is equivalent to (13)–(14); for \( 0 \leq t \leq T \),

\[
(I + r)\frac{1}{2} (\phi(-t) + \psi(t) + h_1(t,0) + h_2(t,0)) \ni \psi(t) + h_2(t,0),
\]

\[
(I + \beta)\frac{1}{2} (-\psi(1+t) + \phi(1-t) + h_1(t,1) - h_2(t,1)) \ni \phi(1-t) + h_1(t,1).
\]

Since \( r \) and \( \beta \) are maximal monotone operators in \( \mathbb{R} \), the functions \( \phi: [-T, 1] \ni \mathbb{R} \) and \( \psi: [0, T + 1] \ni \mathbb{R} \) are uniquely determined.

If \( f_1 \) and \( f_2 \) are smooth enough, (6)–(14) give the classical solution of BVP. However, (6)–(14) make sense under weaker assumptions. Thus \((u, v)\), given by (6)–(14), is called the generalized solution of BVP whenever the integrals in (10)–(11) are well-defined with respect to the Lebesgue measure.

The proofs of the following three lemmas are straightforward; for the proof of Lemma 2 the reader may see [2].

**Lemma 1** Let \( p \in [1, \infty] \), \( u_0, v_0 \in L^p(0, 1) \) and \( f_1, f_2 \in C([0, T]; L^p(0, 1)) \). Then BVP has a unique generalized solution \((u, v) \in C([0, T]; L^p(0, 1))^2\). Moreover, \((u, v)\) does not depend on how \( f_1 \) and \( f_2 \) are extended in \( C([0, T]; L^p_{\text{loc}}(\mathbb{R}))\).

**Lemma 2** Let \( u_0, v_0 \in L^2(0, 1) \) and \( f_1, f_2 \in L^1(0, T; L^2(0, 1))\). Then BVP has a unique weak solution.

**Lemma 3** Assume the conditions of the first Lemma with \( p = 2 \). Then BVP has both the generalized and the weak solution and they coincide.

### 2.1 \( L^2 \)-Valued Solutions

Let \( X = C([0, T]; L^2(0, 1))^2\). Moreover, let \( Y = L^2(\mathbb{R}, 1/(1 + x^4))\), a weighted Hilbert space, and its subspace

\[
U_M = \{ y \in C(\mathbb{R}) \mid y' \text{ exists and } 0 \leq y' \leq 1 \text{ a.e. in } \mathbb{R}, \ |y(0)| \leq M \},
\]

with \( M > 0 \). Consider the mapping \( P: \gamma \mapsto (u_\gamma, v_\gamma) \), where \( r = \gamma^{-1} - I \) and \((u_\gamma, v_\gamma)\) is the generalized solution of BVP. By (13)–(14) and by Ascoli’s theorem we can prove that the mapping \( P: U_M \mapsto X \) is continuous and \( U_M \) is compact in \( Y \), for each \( M > 0 \). Consider the sets

\[
K_M = \{ r \text{ is maximal monotone in } \mathbb{R} \mid |(I + r)^{-1}| \leq M \},
\]

for \( M > 0 \). Clearly, there is a bijection between \( U_M \) and \( K_M \). So, if \((\bar{u}, \bar{v}) \in X\), then by Weierstraß’s theorem there exists \( r_M \in K_M \), minimizing \( J \) in \( K_M \), for any \( M > 0 \).
2.2 Continuous Solutions

Let \( Q_T = ]0, T[ \times ]0, 1[ \) and \( X = C(Q_T)^2 \). Thus we choose a stronger norm in (1). In this case the solution of BVP must belong to \( C(Q_T)^2 \). But that requires \( u_0, v_0 \in C[0, 1] \) and the zeroth order compatibility conditions [1]:

\[
r u_0(0) + v_0(0) \equiv 0, \quad u_0(1) \in \beta(v_0(1)).
\]  

(17)

Hence \( u_0 \) and \( v_0 \) depend on \( r \), and we are guided to define

\[
U = \{ y \in C(\mathbb{R}) \mid 0 \leq y' \leq 1 \text{ a.e. in } \mathbb{R}, \text{ and } u_0(0) = y(u_0(0) - v_0(0)) \}.
\]  

(18)

Again we can easily prove that \( U \) is compact in \( Y \) and \( P : U \mapsto X \) is continuous. Thus there is a maximal monotone operator \( r \subset \mathbb{R} \times \mathbb{R} \) minimizing \( J \) under the constraint (17).

2.3 Continuously Differentiable Solutions

If we choose the norm in (1) to be even stronger, say \( \| \cdot \|_{C^1(Q_T)^2} \), then the things get more complicated: we have to require \( u_0, v_0 \in C^1[0, 1] \), \( f_1 \) and \( f_2 \) to be smoother, and the zeroth and the first order compatibility conditions [1], that is,

\[
(\tilde{r}'(u_0(0) - v_0(0)) - 1)(f_1(0, 0) - v_0'(0)) = \\
= \tilde{r}'(u_0(0) - v_0(0))(f_2(0, 0) - u_0'(0)),
\]  

(19)

\[
(\tilde{\beta}'(u_0(1) + v_0(1)) - 1)(f_1(0, 1) - v_0'(1)) = \\
= \tilde{\beta}'(u_0(1) + v_0(1))(f_2(0, 1) - u_0'(1)),
\]  

(20)

where \( \tilde{r} = (I + r)^{-1} \) and \( \tilde{\beta} = (I + \beta)^{-1} \).

2.4 Finding the Optimal Parameter

The continuity reasoning above establishes only the existence of the optimal \( r \). In order to approximate the optimal parameter \( r \) by the gradient method or to calculate it directly from \( J'(r) = 0 \), we should be able to calculate \( J'(r) \). This leads us to study whether the solution of BVP is differentiable with respect to \( r \).
3 The case of perturbed D’Alembertian

Let $G \neq 0$ or $R \neq 0$. In this case the general solution of (S) can not be expressed by d’Alembert type formulae. However, the problem whether $J$ is continuous and differentiable, reduces to the question of the continuity and differentiability of the fixed point for the mapping $(\xi, \eta) \mapsto (u, v)$, where $(u, v)$ is the solution of (3)–(4) and (21)–(22),

$$\frac{\partial u}{\partial t}(t, x) + \frac{\partial v}{\partial x}(t, x) \in f_1(t, x) - R\xi(t, x), \quad 0 < x < 1, \quad t > 0, \quad (21)$$

$$\frac{\partial v}{\partial t}(t, x) + \frac{\partial u}{\partial x}(t, x) \in f_2(t, x) - G\eta(t, x), \quad 0 < x < 1, \quad t > 0. \quad (22)$$

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