Across the Mediterranean – along the Nile

Volume 1

Studies in Egyptology, Nubiology and Late Antiquity
Dedicated to László Török
on the Occasion of His 75th Birthday

Edited by
Tamás A. Bács, Ádám Bollók and Tivadar Vida

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László Török
(photo: Csaba Villányi and Zalán Péter Salát)

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HIPPARCHUS ON THE RATIO OF LONGEST DAY TO SHORTEST NIGHT IN EUDOXUS, ARATUS AND ATTALUS (*IN ARATI ET EUDOXI PHAENOMENA* I.3.10)

István M. Bodnár

Hipparchus starts his detailed critical commentary about Aratus' Phaenomena *by establishing that Aratus' description of celestial phenomena rests on results presented in Eudoxus'* Phaenomena. *After this, Hipparchus proceeds to engage in a detailed discussion of the contents of Aratus' work, with the major objective of enumerating cases where both Eudoxus and Aratus are in error, thereby also showing that Attalus, the editor and previous commentator of Aratus, was ignorant of celestial matters. This is so, because although he claimed to have established the text of Aratus' didactic poem using as criteria the internal consistency of the text, and also that it should be in conformity with celestial phenomena, he left all these errors standing. As a first instance, Hipparchus turns to Aratus' problematic description of the division of the summer tropic, giving the ratio 5 : 3 between longest day and shortest night in the regions around Greece. In the course of discussion, Hipparchus mentions another value, 12 : 7, also proposed by Eudoxus. Neugebauer suggested an emendation of that value, whereas others – most notably Bowen and Goldstein – argued for retaining the manuscript reading. The paper discusses this debate, signals the difficulties of both options, and argues that the emendation proposed by Neugebauer can be in conformity with what degree of precision Hipparchus expects from astronomers he criticises, and all in all, it makes better sense in the context of Hipparchus' critical argumentation.*

Keywords: Eudoxus; Aratus; Attalus of Rhodes; Hipparchus; Philip (of Opus); latitude; elevation of the pole; celestial observations; critical commentary; use of equinoctial hours; use of degrees; margin of error; degree of precision; Farey sequence

INTRODUCTION

Hipparchus, in his *Commentary on Aratus' and Eudoxus' Phaenomena*, as its title suggests, gives an extended critical discussion of Aratus' *Phaenomena*. First, in Chapter 2 of the first book, he shows, by quoting parallels from Aratus' didactic poem and from Eudoxus' work, that Aratus was dependent on Eudoxus, adding also passages where Aratus reproduced Eudoxus' erroneous statements. The chief antagonist Hipparchus doggedly pursues in the *Commentary*, however, is neither of these earlier authors, but rather Attalus of Rhodes, a contemporary of Hipparchus'. Attalus, like many other people before him, also produced a commentary of Aratus' *Phaenomena*, and was also instrumental in establishing the text of Aratus' poem. He claims in the introduction of his commentary – as quoted by Hipparchus –

> »We therefore have sent you the book of Aratus corrected by us, and its commentary, making everything in it to be in correspondence with what appears in the sky,[1] and to be consistent with what the poet says.« And he continues further: »Perhaps someone might inquire on the basis of what consideration do we say that we effected these corrections in the book according to the

[1] In what follows, I use in my translation the phrase "what appears in the sky" for Hipparchus' τὰ φαινόμενα where it is not the title of Aratus' or Eudoxus' work.

> intention of the author. In reply we provide as the most compelling reason that the poet should be in agreement with what appears in the sky.«[2]

But then, as Hipparchus immediately continues, Attalus by his professed methodological rule exposes himself to a constant barrage of criticisms, because

> […] whatever we shall show from what both Aratus and Eudoxus say, to be contrary to what appears in the sky, we shall have to judge that Attalus also makes the same erroneous assertion about these things.[3]

Indeed, as we shall see, Attalus in these matters will have to be censured more. A prime example of such a case is when Hipparchus claims that both Aratus and Eudoxus were in error about the inclination of the cosmos[4] – or, in our modern terminology, about the latitude of the geographical position from where they make their observations. This is so because the indication Aratus gives – that the summer tropic is divided by the horizon in a ratio of 5 to 3 – would mean that the longest day is 15 hours, the shortest night 9 hours,[5] whereas – Hipparchus hastens to add – this is not true in Greece, but only further north, at the Hellespont. The true value of the longest day in Greece, according to Hipparchus' computations (for which he only indicates the key steps, without setting them out in any detail) is approximately 14 and 3/5 hours.[6]

Aratus is not to blame – being no astronomer himself, he just followed and versified what he found in Eudoxus. And even if he had been responsible for this detail in his didactic poem, unlike Attalus, he does not state where this ratio between longest day and shortest night holds, hence at least in this regard he would avoid censure.[7] But Attalus cannot be so easily let off the hook, because he explicitly – and erroneously – attributes this value to the region of Greece when he writes

> »With this [with the verses that assert that the ratio between the part of the summer tropical circle above the horizon and the part below the horizon is 5 : 3] Aratus makes it clear that he composed the whole work in the regions around Greece, for in these regions is the ratio of longest day to shortest night 5 : 3.«[8]

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[2] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.3: "διὸ δὴ τό τε τοῦ Ἀράτου βιβλίον ἐξαπεστάλκαμέν σοι διωρθωμένον ὑφ᾽ ἡμῶν καὶ τὴν ἐξήγησιν αὐτοῦ, τοῖς τε φαινομένοις ἕκαστα σύμφωνα ποιήσαντες καὶ τοῖς ὑπὸ τοῦ ποιητοῦ γεγραμμένοις ἀκόλουθα." καὶ πάλιν ἑξῆς φησι· "τάχα δέ τινες ἐπιζητήσουσι, τίνι λόγῳ πεισθέντες φαμὲν ἀκολούθως τῇ τοῦ ποιητοῦ προαιρέσει τὴν διόρθωσιν τοῦ βιβλίου πεποιῆσθαι· ἡμεῖς δὲ ἀναγκαιοτάτην αἰτίαν ἀποδίδομεν τὴν τοῦ ποιητοῦ πρὸς τὰ φαινόμενα συμφωνίαν." (Here and elsewhere I follow the text of Hipparchus in the edition of MANITIUS 1894, all translations of Hipparchus, unless indicated otherwise, are mine.)

[3] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.4: τοιαύτην οὖν ἔχοντος τοῦ Ἀττάλου τὴν διάληψιν, ὅσα ἂν ἀποδεικνύωμεν τῶν ὑπὸ τοῦ Ἀράτου καὶ Εὐδόξου κοινῶς λεγομένων διαφωνοῦντα πρὸς τὰ φαινόμενα, δεῖ διαλαμβάνειν καὶ τὸν Ἄτταλον περὶ τῶν αὐτῶν διημαρτημένως συναποφαινόμενον.

[4] ἔγκλιμα τοῦ κόσμου, or also the elevation of the celestial pole (ἔξαρμα τοῦ πόλου), i.e. the angle between the horizon and the straight line to the celestial pole.

[5] In Hipparchus' terminology: 15 *equinoctial* hours. I drop this added clarification, as all the references in this paper will be to equinoctial hours of uniform length.

[6] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.5–7, on Hipparchus on shadow lengths and longest days, see NEUGEBAUER 1975, 746–747, esp. the following: "Though mathematically correct this result nevertheless does not agree very well with the actual latitude of Athens which is 38°. Again we see that the ratio 4:3 [between the length of the *gnōmōn* and the shadow it casts at noon on the day of the summer solstice] is only a convenient estimate and not the result of careful observation. Such primitive data were apparently never checked: at least Vitruvius, some 150 years later, still quotes the same ratio."

[7] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.8 – note that Hipparchus' exonerating remark is tentative, and restricted to the status of this detail.

[8] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.9: "διὰ δὲ τούτου φανερὸν ποιεῖ, διότι τὴν ὅλην πραγματείαν ἐν τοῖς περὶ τὴν Ἑλλάδα τόποις πεποίηται· παρ᾽ ἐκείνοις γάρ ἐστιν ἡ μακροτάτη ἡμέρα πρὸς τὴν μικροτάτην νύκτα ὡς τὰ ε΄ πρὸς τὰ γ΄."

Attalus, as Hipparchus hastens to add, further compounds his error by surprisingly not taking into account

> that Eudoxus sets (the matter) out differently in his other work and writes that the ratio between the section of the tropic above the Earth and the section below the Earth is 12 : 7, and that the Philippans and many others alike make a record in a way similar to this, except that they have arranged the simultaneous risings and settings of the stars as observed in the regions around Greece, but have been in error concerning the inclination of these regions.[9]

There are several things we can immediately take away from these assertions. One is that Attalus should have realised the discrepancy between the Eudoxan works. This is even more pressing, because – as Hipparchus has shown in the previous chapter[10] – Aratus follows Eudoxus' *Phaenomena*, whereas in this particular instance, the value is taken from his other work, the *Mirror* (Ἔνοπτρον).[11] Aratus may have had all sorts of reasons for preferring this value. Attalus, however, should have paused to consider what to make of this discrepancy between Aratus on the one hand, and Eudoxus' *Phaenomena* and a number of further astronomical authorities, including a certain Philip, most probably Philip of Opus, on the other.[12] Moreover, the mention of the Philippans and some further astronomers also introduces a contrast. This time between them, and Eudoxus' *Phaenomena.* These astronomers adopted the erroneous value for the ratio of longest day and shortest night from Eudoxus' *Phaenomena.* But they – unlike Aratus – did not adopt the problematic further astronomical details from this Eudoxan work. Their astronomical observational records are summarily accepted by Hipparchus as appropriate for the regions around Greece.

Once Hipparchus has established that Aratus' value for the ratio between longest day and shortest night is in error, one could level criticisms against Aratus' further assertions from two different directions. One would be to check whether what we find in Aratus is in correspondence with what can be observed from Greece, whereas the other would be to check internal consistency, and see whether the observations described by Aratus are correct for those regions where the ratio between longest day and shortest night is 5 : 3. In closing his discussion about the true value of the ratio of longest day and shortest night, and before turning to a detailed critical discussion of Aratus' description of celestial phenomena, Hipparchus makes it clear that he is not interested in the second line of criticism:

> Leaving this error aside, we have investigated the whole of their composition as against the horizon in Greece. For it is not appropriate for someone aiming at truth, but rather for someone zealous about frivolities, to attack them in each and every instance that is in contradiction with

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[9] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.10: ἔτι δὲ μᾶλλον θαυμάσειεν ἄν τις, πῶς ποτε οὐκ ἐπέστησε τοῦ Εὐδόξου ἐν τῷ ἑτέρῳ συντάγματι διαφόρως ἐκθεμένου καὶ γράφοντος, ὅτι τὸ ὑπὲρ γῆν τοῦ τροπικοῦ τμῆμα πρὸς τὸ ὑπὸ γῆν λόγον ἔχει, ὃν <ἔχει> τὰ ιβ΄ πρὸς τὰ ζ΄, ὁμοίως δὲ τούτῳ καὶ τῶν περὶ Φίλιππον ἀναγραφόντων καὶ ἄλλων πλειόνων, πλὴν ὅτι συντετάχασι μὲν τὰς συνανατολάς τε καὶ συγκαταδύσεις τῶν ἄστρων ὡς ἐν τοῖς περὶ τὴν Ἑλλάδα τόποις τετηρημένων, κατὰ δὲ τὸ ἔγκλιμα τῶν τόπων τούτων διημαρτήκασι.

[10] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.2.

[11] This is a detail Hipparchus mentioned just before embarking on his criticism of Aratus' value of the ratio of longest day and shortest night, after the list of correspondences between Aratus' and Eudoxus' *Phaenomena*. He signals that this time he is calling attention to a correspondence between Aratus and Eudoxus' *Mirror*: "Apart from these, Aratus supposes the inclination of the cosmos <to be the same> as Eudoxus, for Eudoxus, too, asserts in the *Mirror* that the tropic is divided so that the segments are in the ratio 5 : 3 to each other." (Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.2.22: Χωρὶς δὲ τούτων καὶ τὸ κλίμα τοῦ κόσμου <τὸ αὐτὸ> τῷ Εὐδόξῳ ὑποτίθεται ὁ Ἄρατος. καὶ γὰρ ὁ Εὔδοξος ἐν τῷ ἐπιγραφομένῳ Ἐνόπτρῳ τὸν τροπικὸν τέμνεσθαί φησιν οὕτως, ὥστε λόγον ἔχειν τὰ τμήματα πρὸς ἄλληλα τὸν αὐτόν, οἷον ἔχει τὰ ε΄ πρὸς τὰ γ΄.)

[12] On Philip, see NEUGEBAUER 1975, 739–740, esp. n. 12, and 929.

> their discredited assumption, even when these are said in accordance with what is seen in Greece in the sky.[13]

An Emendation and its Discontents

One could have thought that the case I have presented so far is straightforward, and sufficiently clear. Eudoxus in two different works propounded two different values for the ratio between longest day and shortest night, both of them problematic. Aratus took one of them, whereas many subsequent astronomers took the other one. Otto Neugebauer, however, in his magisterial *A history of ancient mathematical astronomy* called attention to the fact that the ratio 12 : 7 between longest day and shortest night would imply a division of a circle into 19 equal parts, producing a value for the length of the day that cannot be expressed in unit of hours, or segments of the tropic that cannot be expressed in degrees, and suggested that the text should be emended to read 11 : 7 instead.[14] A further advantage of this emendation might be that the ratio 11 : 7 between longest day and shortest night is attested by Pliny for Athens. So, as Neugebauer also adds, with this emendation the passage would give Eudoxan pedigree to this value on Pliny's list.[15]

But Neugebauer's suggestion did not remain unchallenged in the literature. In an important paper, Alan Bowen and Bernard Goldstein argued that the reading of the manuscripts at Hipparchus' *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.7 should be retained. First, they argue, the use of equinoctial hours is not attested by the time of Eudoxus.[16] Moreover, all the known traditional values of ratios of longest day to shortest night can be assigned on a sequence of ratios, which is generated by a simple numerical rule: take 1 : 1 and 2 : 1 as the two initial limiting values (the ratios at the Equator and what a Greek would consider somewhere unbearably far north, respectively), interpolate (a + c) : (b + d) between a : b and c : d, and then we shall have a series comprising of all the traditional values for this fundamental ratio, characterising the geographical latitude of different locations as presented in *Table 1* (indicating in square brackets at which step the value was introduced in the series).

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[13] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.11: Παραπέμψαντες <οὖν> τοῦτο τὸ ἀγνόημα τὴν ὅλην αὐτῶν σύνταξιν ἐπεσκεψάμεθα πρὸς τὸν ἐν τῇ Ἑλλάδι ὁρίζοντα. οὐδὲ γὰρ φιλαλήθους, ἀλλὰ κενοσπούδου, τὸ κατὰ πάντα μαχόμενον τῇ διεψευσμένῃ ὑποθέσει ἐπιλαμβάνεσθαι αὐτῶν, κἂν τύχῃ συμφώνως λεγόμενα τοῖς ἐν τῇ Ἑλλάδι φαινομένοις.
For a case when Hipparchus nevertheless uses an argument of the second kind, see I.7.19–22: Here Aratus (and Attalus) are censured for Aratus' assertion that the constellation Cepheus rises and sets from the head all the way to its belt. Criticising this statement, Hipparchus sets out that from Greece only a smaller part – from head to shoulders – rises and sets, the rest is circumpolar (I.7.19–21). Then he continues with the remark that "the error turns out to be much greater if we assumed the inclination of the world he himself proposes: for where the longest day is 15 hours, there the whole of the constellation Cepheus is circumpolar" (πολλῷ δὲ μεῖζον γίνεται τὸ ἀγνόημα, κἂν ὑποθώμεθα τὸ καθ᾽ ἑαυτὸ<ν> ἔγκλιμα τοῦ κόσμου· ὅπου γὰρ ἡ μεγίστη ἡμέρα ὡρῶν ἐστι ιε΄, ἐκεῖ ὅλος ὁ Κηφεὺς ἐν τῷ ἀρκτικῷ φέρεται. I.7.22, where I read καθ᾽ ἑαυτὸν instead of καθ᾽ ἑαυτὸ of family A of the manuscripts, along the lines of the emendation by Scaliger, who reads τὸ κατ᾽ αὐτὸν ἔγκλιμα). Note, however, that this is not in violation of the limits of criticism as announced in I.3.11. It is only after arguing that the assertion of Aratus is not borne out by what can be seen in the sky from Greece that Hipparchus launches this further, even more devastating objection.

[14] Neugebauer 1975, 733 n. 28.

[15] Indeed, Neugebauer 1975, 733 n. 28, makes the even more momentous suggestion that with this emended value "we would have the earliest evidence for the arithmetical climata," laid out on Neugebauer 1975, 730.

[16] See Bowen–Goldstein 1991, 240 (in § 3: The earliest occurrence of equinoctial hours and time-degrees in Greek astronomy), arguing that "there is no unambiguous evidence in Greek texts […] for the use of equinoctial hours until P. Hibeh 27, or for the use of seasonal hours until Timocharis […]".

1:1 [1]	3:2 [2]	5:3 [3]	12:7 [5] {7:4 [4]}	2:1 [1]

Table 1

There are several problems with this proposal. One is that the series presented by Bowen and Goldstein is not uniformly interpolated – after the first interpolation of the value 3 : 2, first only the right limb is interpolated (5 : 3, but not 4 : 3), then again only the right one (7 : 4, but not 8 : 5), and finally only the left one (12 : 7, and not 9 : 5). One should expect some explanation for this sequence of interpolations, and also how it came about that a particular value was assigned to a particular locality, especially in view of the fact that Bowen and Goldstein put forward the proposal in order to show that "it would have been possible for Eudoxus to obtain the ratios 5 : 3 and 12 : 7 without recourse to measurement or observation."[17]

The other consideration which tells against such a sequence of numerical interpolations is that the data is slightly doctored for better fit: the value Bowen and Goldstein assign to Byzantium, 7 : 4, which I enclosed in braces on the list, is not attested explicitly as a traditional value for a ratio of longest day and shortest night,[18] and at least two important values are missing: 7 : 5 for Alexandria, or Lower Egypt (attested in various ways, in Hibeh Papyrus 27 col. viii 115–120 presumably for Sais, in the *Ars Eudoxi*, col. ii 22–25 and col. iii 1–5, without explicit location, in Hypsicles *Anaphoricus* 63–66 and in Pliny *Historia naturalis* VI.212, for Alexandria), and 11 : 7, for Athens (attested by Pliny *Historia naturalis* VI.215). The resulting list, including these values and suppressing the ratios 12 : 7 and 7 : 4, can again be arranged in a sequence of numerical interpolations, as in *Table 2* again with the addition of the ratios 4 : 3 and 8 : 5 in braces.

1:1 [1]	{4:3 [3]}	7:5 [4]	3:2 [2]	11:7 [5] {8:5 [4]}	5:3 [3]	2:1 [1]

Table 2

This, however, should come as no surprise, this is just a consequence of the fact that sequences of ratios generated with this rule of interpolation – so-called Farey sequences – will reach every ratio (for the proof, see the Appendix). Consequently, a sequence of such interpolations could be of even less use as a heuristic device in the process of determining what ratio to assign to a particular location.

This means that if we want to decide whether the value transmitted in the manuscripts for the ratio of longest day to shortest night in Eudoxus' *Phaenomena* should be emended in Hipparchus, text or not, this needs to be discussed with two foremost considerations in mind: with the overall coherence of what other testimony we have about the methods, means and results of Eudoxan astronomy, and with the coherence of the particular context in which the value is transmitted. The first line of considerations is not likely to produce uncontested results: it is not clear whether equinoctial hours, let alone any canonical division of the day for astronomical purposes would already have been in use by Eudoxus' time, and it is bound to remain a matter of contention whether the values 7 : 5 (for Alexandria and environs) and 11 : 7 (for Athens) on a composite list of Pliny's, apparently drawing on many disparate sources, rest on the authority of Eudoxus himself.

[17] Bowen–Goldstein 1991, 238.

[18] Bowen–Goldstein 1991, 237, Table 1, refer to Strabo II.5.41 for this value. But what that passage claims is that in the regions of Byzantium, the longest day is 15 ¼ hours, giving a shortest night of 8 ¾ hours. This would result in a ratio of 61 : 35. In explanation, Bowen–Goldstein 1991, 237 n. 9, repeat the suggestion of Goldstein 1983, 10–12, that this value for longest day may well go back to Hipparchus, who in working out the values of his list of longest days *vs* shortest nights on which Strabo's list is based could probably start out from the ratio 7 : 4 for Byzantium, and then adjusted it to the nearest quarter hour. On the other hand, as we have seen, Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.7 assigns the ratio 5 : 3 to the Hellespont, close to the location of Byzantium.

Hence I suggest that we need to assess the case from an evaluation of the passage itself, and see what sense it makes with the transmitted value for the ratio of longest day and shortest night, and whether emending this value might make better sense.

Points of Concern about the Transmitted Value, and about the Value as Emended by Neugebauer

There are two crucial considerations against the transmitted value. First, it is not clear why Philip, and the many astronomers following his example should have adopted from among the two ratios the second one, 12 : 7. This is so, because it would result in a value of 15 and 3/19 hours for the longest day, pushing the regions around Greece even further to the north than the other ratio, of 5 : 3. Aratus' example suggests that this other value of 5 : 3 was available in the *Mirror*, and could be transferred into the context of the work that rested on Eudoxus' *Phaenomena.* Moreover, if Attalus took the admittedly incorrect ratio of 5 : 3, which by Hipparchus' lights produces a longest day 2/5 hours too long, but which nevertheless is somewhat better than what Eudoxus used elsewhere, Hipparchus' remark is quite strange when he says that

> one would be even more surprised how on earth Attalus did not realise that Eudoxus set (the matter) out differently in his other work when he writes […].[19]

It is not clear at all why in the context of Attalus' acceptance of an erroneous value from Aratus, and ultimately from Eudoxus the fact should cause consternation that he did not realise that in another work of Eudoxus an even more problematic value was available.

Perhaps one might wish to take this objection as a not so blunt way on the part of Hipparchus to insinuate that Attalus was working only with an eye to Aratus' text, without recourse to its main source (according to Hipparchus), the *Phaenomena* of Eudoxus. This in addition to that other insinuation, which is doubtless present in Hipparchus' words in the following clause, that Attalus was not aware of the results of "the Philippans and many other astronomers," who "have arranged the simultaneous risings and settings of the stars as observed in the regions around Greece",[20] the main topic in which Hipparchus will take issue with Attalus in the course of the commentary. Note, however, that the commentary itself will not follow this path. Attalus is not going to be censured because he did not follow this or that correct observation of some earlier astronomical authority in establishing and explicating Aratus' text. The sole standard that he will be held to is the one he professed to follow in his introduction – that the assertions of the poet should be in "agreement with what appears in the sky".[21] This requires astronomical knowledge on his part, and this is what Hipparchus will show, again and again, that he lacks. No amount of familiarity with different values present in astronomical literature can be a substitute for such expertise in astronomy. In accordance with this, the fact that Attalus did not recognise an even more problematic value than the one he left standing in Aratus' text would not be relevant in this regard.[22] As far as agreement

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[19] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.10: ἔτι δὲ μᾶλλον θαυμάσειεν ἄν τις, πῶς ποτε οὐκ ἐπέστησε τοῦ Εὐδόξου ἐν τῷ ἑτέρῳ συντάγματι διαφόρως ἐκθεμένου καὶ γράφοντος […].

[20] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.10.

[21] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.3.

[22] Cf. the exposition in Lasserre 1966, 194–195, according to which the ratio 12 : 7 in the earlier *Phaenomena* is a fundamentally flawed value for Cnidus, whereas the ratio 5 : 3 is a rather good approximation for the ratio of longest day and shortest night for Cyzicus in the later *Mirror*.
Lasserre's proposal to the difficulties of this passage, then, includes the following claims: Philip of Opus accepted the value 12 : 7 for the proportion of longest day to shortest night from the *Phaenomena*, because the later *Mirror* had not been available to him. Other astronomers, as Hipparchus attests, followed his example – unlike Aratus, who accepted the value 5 : 3 from the later *Mirror*. This neat scheme, however, does not explain why Hipparchus should criticise Attalus for not taking the fundamentally flawed value of the earlier work on board.

with celestial phenomena is concerned, this other value, at best, can serve as a spur, suggesting that the issue should have required fresh investigation on Attalus' part.

All in all, Hipparchus' text here would have far greater argumentative appeal if one could introduce an emendation and include a ratio that by Hipparchus' lights is still incorrect, but is closer to the one accepted by him. One such emendation could be 11 : 7, as proposed by Neugebauer. But neither is this value unproblematic. Notably, it is *too* close to the ratio given by Hipparchus.[23] Hipparchus establishes that the longest day is 14 and 3/5 hours (14 hours and 36 minutes), whereas the emended ratio, 11 : 7, gives a longest day of 14 and 2/3 hours (14 hours and 40 minutes). The difference is just 4 minutes – or if one were to speak about the segments of the circle of the summer tropic, as Aratus and Eudoxus formulate their claim, the difference is altogether one degree, 1/360 part of the whole circle. With this minute difference one needs to raise the question not only whether it is fair on Hipparchus' part, but whether it is at all credible that he should rebuke Philip, and many other astronomers that although on the whole their account is in accordance with celestial phenomena as observed from Greece, they nevertheless are in error, because their division of the tropical circle into segments corresponding to longest day and shortest night is one degree off the mark.[24]

Degrees of Precision, and a Tentative Proposal

One way of trying to formulate an at least tentative proposal could be to engage in a systematic study of what degree of precision Hipparchus expects from his predecessors. This is not easy to calibrate, as Eudoxus and Aratus do not express their claims in straightforward quantitative terms; instead, most often they give descriptions of the configurations of the stars. In rebutting these, Hipparchus may or may not use exactly expressed values, but either way his criticism is usually targeted at discrepancies of considerable size.

Nevertheless, there are some passages where the error which Hipparchus attributes to his predecessors could be quite small. One such instance might be where Hipparchus mentions at I.2.11–12 and at I.5.19 Eudoxus' and Aratus' claim that the two feet of Cepheus (γ and κ Cephei) and the tail of the Small Bear (α Ursae minoris, today's Pole Star) form an equilateral triangle. This claim is chastised by Hipparchus on both occasions: the distance between the two feet is smaller than the other two distances, and as he adds in I.5.19, this will make the triangle isosceles and not equilateral. In this case, the difference between the base and the shorter of these "legs" of the triangle is slightly above one degree, and there is also a difference of half a degree between the two putatively equal legs of the triangle. Accordingly, this case might suggests that in cases of directly observable magnitudes, an error of one degree (between distances of c. 11 and 12 degrees) is already significant, whereas the error of half a degree, the difference between the two putatively equal sides of this triangle, is still tolerable.

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[23] This consideration could give rise to alternative suggestions, e.g. emending the ratio in the text, 12 : 7 into 13 : 8 or even to 14 : 9. These more convenient, "corrected" values, however, are just as much unattested in the ancient record as the reading of the manuscripts, 12 : 7 is.

[24] It can also be illuminating to take into account that as Lloyd 1982, 143–144, stresses "ancient astronomers could tell the time at night to an accuracy of within ten minutes, which will correspond to between two and three degrees in the motion of the stars on the celestial equator. In line with this, no actual recorded observation in Ptolemy is more precise than to within one-sixth of an hour." Accordingly, the differences of these lengths of the longest days (or of the shortest nights) are all computed – or otherwise derived – values which could not be ascertained by direct observation. ("Otherwise derived": in principle, such details could be read off from suitably constructed representations or diagrams. Cf. Neugebauer 1975, 279, who remarks that "[Hipparchus'] way of determining the position of a star [in the second part of the *Commentary*] is convenient both for readings on a globe and for graphic construction or plane trigonometric computation based on stereographic projection […], assuming that the latter was known to Hipparchus.")

However, this line of thought rests on the assumption that we assess the error Hipparchus attributes to his predecessors with the help of actual, direct sightings. One might argue that the distances Hipparchus has in mind need not be directly observed or measured, but may be derived from values for the location of these stars that Hipparchus started out from. Unfortunately, the information we have about such values is by no means complete. From the indications in Ptolemy's *Almagest* and Hipparchus' *Commentary*, Vogt computed the the position of α Ursae minoris according to Hipparchus' catalogue of fixed stars,[25] but for the other two stars – γ and κ Cephei – we have only partial indications from the second half of Hipparchus' *Commentary*, describing the risings and settings of some constellations.[26] Cepheus, being a mostly circumpolar constellation is not among the constellations described there. Accordingly, for these stars the degree of the ecliptic they are culminating together with is given as happening together with the beginning or the end of the culmination of some other constellation. This happens, for both of them, on two occasions, indeed, with some slight difference: γ Cephei is described first as culminating together with the beginning of the culmination of Lepus, together with the 4th degree of Pisces,[27] and then as culminating together with the end of the culmination of Sagittarius, together with the middle of the 5th degree of Pisces,[28] whereas κ Cephei is described first as culminating together with the beginning of the culmination of Serpens, together with the 8th degree of Aquarius, and then as culminating together with the beginning of the culmination of Orion, together with the 9th degree of Aquarius.

We can take away from these indications that culminations, in the core cases, were meant to be recorded with a precision of half a degree.[29] At the same time, however, culminations of neighbouring stars could get indicated with a tolerance of error approaching one degree. This suggests that the size of error tolerated may be flexible: it may depend on several factors, one among them whether the phenomenon mentioned is the one discussed directly, or it is a concomitant to the core phenomenon being discussed.[30] This somewhat adjustable degree of precision on Hipparchus'

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[25] VOGT 1925. For a criticism of the unannounced assumptions on which Vogt's computations rest, see GRASSHOFF 1990, 104–107 (Sections 4.2.2 Reconstruction of Coordinates and 4.2.3 The Accuracy of the Reconstructed Coordinates).

[26] For a discussion of Hipparchus' indications in the second part of the *Commentary*, see GRASSHOFF 1990, 174–197, and Appendix C.

[27] Note that the 4th degree is 3 degrees from the beginning of the constellation (on this see NEUGEBAUER 1975, 279).

[28] The middle of the 5th degree is half a degree added to the 5th degree, i.e. 4.5 degrees from the beginning of the constellation.

[29] It is about these stars that Hipparchus claims that he "will set out each of these details by way of approximation up to the point of insignificant difference." (Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* II.4.6: ἕκαστον δὲ τούτων διασαφήσομεν κατὰ συνεγγισμὸν ἕως ἀδιαφόρου παραλλαγῆς, translation from GRASSHOFF 1990, 106, slightly modified. Note that Graßhoff takes this claim to apply to all the values in the second part of the *Commentary* – both to those of the core cases of the constellations discussed, and those of the culminations of the neighbouring stars. As a result, he summarily claims that "A careful astronomer like Hipparchus could never have failed to realize when the same phenomenon, […], is recorded in different places of the book with different values – unless he used a globe which is not capable of precise reproduction of the phenomena. […] [Vogt's reconstructed stellar coordinates] are founded upon values which have only an indirect relationship to the Hipparchan fixed star register." GRASSHOFF 1990, 106f. For a different assessment of the effects of Hipparchus' using a mobile sphere, see NADAL–BRUNET 1984.)

[30] Note that Hipparchus' indications can convey even more precision when he sets up the lists of fixed stars that are at equal, hourly distances from one another in the sky. In these lists he includes stars which are not exactly on the meridian in question, but deviate from it 1/10, 1/20, or 1/30 of an hour. This procedure again suggests a sliding scale of precision. Further away from the meridian, the calibration is 1/20 of an hour, corresponding to ¾ degree on the daily circuit of the star, whereas in those cases where the star is indicated to be at a distance of 1/30 hour, as contrasted to 1/20 hour, the difference between these values is 1/60 of an hour, corresponding to ¼ degree on the daily circuit of the star.
Furthermore see the description of the second hourly circle, where Hipparchus says "this little star precedes

part, however, does not take us closer to figuring out what degree of precision Hipparchus expects from Eudoxus and Aratus when he criticises their claim that γ and κ Cephei and α Ursae minoris form an equilateral triangle. Neither do we know how great he took the difference to be between the base of the triangle and its legs, nor can we determine what degree of precision he should expect in such a case.

In the case of another not directly observable magnitude, the margin of error tolerated is definitely smaller than half a degree. Chapter 9 of Book I is devoted to a discusssion of whether the celestial equator, the two tropical circles and the zodiacal circle are treated by Aratus and the astronomers as geometrical circles, without breadth, or as extended celestial bands. The discussion is introduced by a remark on the text of Aratus: Hipparchus quotes lines 467f., where he reports that some manuscripts read "these [the circles] are without breadth and fastened all to each other […]", whereas others have "these are broad, and fastened all to each other […]".[31] Hipparchus criticises Attalus for opting for the second alternative, and adding the claim that

> »the astronomers, too, assume that the tropical circles and the [celestial] equator and the zodiacal circle have breadth, because the sun does not always make its turn on the same circle, but sometimes further south, sometimes further north.«[32]

Hipparchus admits that Eudoxus in the *Mirror* indeed asserts »that the turns of the sun happen at different points, albeit the difference is very small and hard to observe«,[33] but he goes on to argue against the assumption of celestial circles having breadth nevertheless. The key consideration Hipparchus adduces in this argument is that if the sun's course deviated from the ecliptic, this would affect the eclipses of the moon, which are computed from the assumption that the sun's path follows exactly the ecliptic, resulting in computations of lunar occultations with a degree of precision within two digits. "To say that the circles, too, have this breadth [of two digits] amounts to assuming that they are without breadth,"[34] Hipparchus concludes, before adding that it is not clear whether this error of two digits is the result of the discrepancy in the sun's or in the moon's movement.

This tolerance of an error of two digits – i.e. ten minutes of arc, roughly the third of the diameter of the moon – also suggests that an error of the magnitude of a whole degree, twice the diameter of the moon, would be not so easy to accommodate.

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with less than a digit the circle through the poles which marks the second hourly distance," a digit being five minutes of arc. (Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* III.5.2 οὐδὲ δάκτυλον προηγεῖται οὗτος ὁ ἀστερίσκος τοῦ διὰ τῶν πόλων κύκλου τὸ δεύτερον ὡριαῖον διάστημα ἀφορίζοντος.)

[31] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.9.1, translations based on the translation on KIDD 1997, 107. The variants are αὐτοὶ δ᾽ ἀπλατέες καὶ ἀρηρότες ἀλλήλοισι | πάντες or αὐτοὶ δὲ πλατέες καὶ ἀρηρότες ἀλλήλοισι | πάντες. The Loeb edition of G. R. Mair (based on the text in the manuscript tradition, as reported by him and by Kidd in his critical apparatus, and on the *Aratus Latinus*, the eighth-century Latin translation) reads αὐτοὶ δ᾽ ἀπλανέες καὶ ἀρηρότες ἀλλήλοισι | πάντες, translating "the circles are immovable, and fitted each to other" (MAIR 1960, 244f.). KIDD 1997, 106 accepts Hipparchus' reading, ἀπλατέες, and claims (KIDD 1997, 349f) that it forms the basis of the scholia in the manuscripts, even though they feature as explicating the reading ἀπλανέες there. (For the text of the scholium, see MARTIN 1974, 296.)

[32] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.9.1: "καὶ γὰρ οἱ ἀστρολόγοι, φησί, πλατεῖς ὑποτίθενται τούς τε τροπικοὺς καὶ τὸν ἰσημερινὸν καὶ τὸν ζῳδιακὸν διὰ τὸ τὸν ἥλιον τὰς τροπὰς μὴ ἀεὶ ἐπὶ τοῦ αὐτοῦ κύκλου ποιεῖσθαι, ἀλλὰ ποτὲ μὲν νοτιώτερον, ποτὲ δὲ βορειότερον."

[33] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.9.2: "φαίνεται δὲ διαφορὰν τῶν κατὰ τὰς τροπὰς τόπων καὶ ὁ ἥλιος ποιούμενος, ἀδηλοτέραν δὲ πολλῷ καὶ παντελῶς ὀλίγην."

[34] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.9.5: τό γε μὴν λέγειν τηλικοῦτον πλάτος ἔχειν καὶ τοὺς κύκλους ἴσον ἐστὶ τῷ ἀπλατεῖς αὐτοὺς ὑποτίθεσθαι, […]. With this tolerance of error cf. the precision Hipparchus uses in locating the little star in the vicinity of the second hourly circle, see note 30 above.

This case, however, does not directly address the issue whether a difference of a single degree between Hipparchus' value for the arc of the summer tropic above the horizon in Greece and the one corresponding to the emendation, that the ratio of longest day to shortest night is 11 : 7, is sufficient for Hipparchus to maintain that astronomers who propounded this value are in error. We have seen now that Hipparchus can claim that quite small discrepancies are errors. But it should be borne in mind that we have also seen that different cases call for differences in their tolerance of error. The fact that the claim about the breadth of celestial circles does not allow for deviations of a degree does not guarantee that when we speak about the division of the tropical circle by the horizon, a difference of a degree would also consitute an error worthy of repeated criticism by Hipparchus.

Accordingly, I need to tackle a case that is expressly about the length of the longest day. For this we need to turn to the second half of the *Commentary on Eudoxus' and Aratus' Phaenomena*, which is actually not any longer a commentary to the *Phaenomena* of these authors, but a description of the simultaneous risings and settings of the fixed starts "according to truth".[35] Hipparchus, however, characterises the "regions around Greece" where these risings and settings are recorded differently from what we read about the regions around Greece in the discussion of I.3.10. There the longest day was said to be 14 and 3/5 hours, whereas the observations of this second part of the work are valid for "the regions around Greece, and in general [for regions] where the longest day is 14 ½ hours long."[36] This, however, should not be taken as an indication of tolerating an error of six minutes in the length of the longest day, corresponding to 1.5 degrees on the segment of the tropical circle above the horizon. Instead, what this different value indicates is that the second part gives a description of the celestial phenomena as observed from Rhodes, at latitude 36 degrees, whereas the first part professedly criticised Eudoxus and Aratus on the basis of taking a location at approximately 37 degrees latitude, i.e. Athens as the place of observations.[37]

Indeed, just after setting out what he intends to accomplish in this second part he adds that "the individual proofs pertaining to these we have compiled elsewhere in a way so that in practically every region of the inhabited world one can trace accurately the differences of the simultaneous risings and settings".[38]

This is important, because this suggests that Hipparchus does by no means suggest that the difference between the risings and settings at these places, Athens and Rhodes, and also between the lengths of the longest days would be negligible, in the range of error that can be tolerated in such matters. Instead, when he starts out in the second part to describe the simultaneous risings

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[35] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.1.9: "I have described for you according to truth the simultaneous risings and settings of each of the fixed stars together with the twelve zodiacal constellations themselves" (ἀναγέγραφά σοι καὶ τὰς κατὰ ἀλήθειαν γινομένας ἑκάστου τῶν ἀπλανῶν ἄστρων σὺν αὐτοῖς τοῖς κατηστερισμένοις δώδεκα ζῳδίοις συνανατολὰς καὶ συγκαταδύσεις,[…]).

[36] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* II.4.2: ἐν τοῖς περὶ τὴν Ἑλλάδα τόποις καὶ καθόλου ὅπου ἐστὶν ἡ μεγίστη ἡμέρα ὡρῶν ἰσημερινῶν ιδ΄ καὶ ἡμιωρίου.

[37] For the statement that the first part speaks about celestial phenomena as observed from Athens at latitude c. 37 degrees, see Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* I.3.12. In I.7.21 and I.11.8 Hipparchus contrasts Athens to a place where the latitude is 36 degrees, in the latter instance also indicating that this is the location of Rhodes, where Canopus, contrary to Eudoxus' claim, is observed.
In accordance with this distinction, I suggest that in those two instances in the first, commentary part (I.7.11 and I.7.14) where Hipparchus makes claims about "the regions around Greece and where the longest day is 14 ½ hours long" the conjunction καί is copulative and not epexegetic. Indeed, in the second case, Manitius introduces the conjunction καί as a conjecture before the clause beginning with ὅπου precisely in order to avoid the characterisation of the regions around Greece that the longest day would be 14 ½ hours long there, as his translation of the passage unambiguously shows.

[38] Hipparchus, *Commentary on Aratus' and Eudoxus' Phaenomena* II.4.3: τὰς δὲ κατὰ μέρος αὐτῶν ἀποδείξεις ἐν ἄλλοις συντετάχαμεν οὕτως, ὥστε ἐν παντὶ τόπῳ σχεδὸν τῆς οἰκουμένης δύνασθαι παρακολουθεῖν ταῖς διαφοραῖς τῶν συνανατολῶν καὶ συγκαταδύσεων.

and settings from Rhodes, he actually stresses that this can be used as the basis for finding the corresponding data about these phenomena as observed from a different location.

This, then, allows that just as he makes a clear and systematic distinction between the phenomena as observed from Rhodes and from Athens, he can have a similar reason to distinguish between a place where the longest day is 14 hours and 36 minutes long, and another one where the longest day is 4 minutes longer. Otherwise put: Hipparchus may find it vindicated to charge astronomers with an error if they use this latter value in place of the former.

All this, then, shows that Neugebauer's emendation is perfectly admissible. As a result, the considerations of Section 3 above – where I argued that on the reading of the manuscripts, with the value 12 : 7, Hipparchus' argumentation will be problematic – establish the conclusion that once Neugebauer's emendation is admissible, it is also clearly preferable to the problematic reading of the manuscripts.

Appendix

In order to prove that the sequence of interpolations described by Bowen and Goldstein will accommodate every possible ratio within the initial range {1 : 1, 2 : 1} first we introduce the relation "being in close sequence" between ratios, defined as follows.[39] Let a : b and c : d be two ratios, both of them expressed in further not reducible form, i.e. where there is no integer number larger than 1 that a and b, or c and d would be the multiples of this number, respectively. Then we say that a : b and c : d are in *close sequence* iff b * c = a * d + 1.[40] In this case, then, there is an integer p so that the compound of the ratios b : a and c : d is identical to a ratio of the form p + 1 : p.

Furthermore, if a : b and c : d are in close sequence, then a : b will be in close sequence with the interpolated (a + c) : (b + d),[41] since b * (a + c) = a * (b + d) + 1, and this interpolated ratio, in turn is going to be in close sequence with c : d, since (b + d) * c = (a + c) * d + 1.

As a next step, it can be shown that between two ratios, a : b and c : d, that are in close sequence, one cannot interpolate a ratio u : v so that v < b + d. As we have seen, the ratio (a + c) : (b + d) is between a : c and b : d, being pairwise in close sequence with the two original ratios. Then suppose that there is a ratio u : v to be found, where v < b + d, so that a : b < u : v < (a + c) : (b + d). In this case, we shall compare the two compounds, the one of b : a and (a + c) : (b + d), and the other of b : a and u : v. The first one is equal to b * (a + c) : a * (b + d), where, by the fact that a : b is in close sequence with (a + c) : (b + d), b * (a + c) = a * (b + d) + 1.

The compound of b : a and u : v, equal to b * u : a * v should be smaller than this compound, if u : v is to be smaller than (a + c) : (b + d). This, however, cannot be the case, because – by assumption – v should be smaller than b + d, and so b * u : a * v should exceed the ratio a * (b + d) + 1 : a * (b + d), on the assumption that a : b < u : v < (a + c) : (b + d). But then we arrived from the supposition that u : v is smaller than (a + c) : (b + d) to the consequence that it has to be larger than it. The same line of reasoning – starting out from the compounds of (a + c) : (b + d) and d : c, and of u : v and d : c, respectively – applies to the case under the supposition that (a + c) : (b + d) < u : v < c : d, with the upshot that in this case, again contrary to our initial supposition, the ratio u : v will have to be smaller than (a + c) : (b + d). This is so, because (a + c) : (b + d) is in close sequence with c : d, and so (b + d) * c = = (a + c) * d + 1, which makes (a + c) * d : (b + d) * c identical to (b + d) * c - 1 : (b + d) * c. This, however, will be larger than the compound u * d : v * c, because v < b + d, and u * d : v * c < 1 : 1. Hence between two ratios, a : b and c : d that are in close sequence, one cannot insert a ratio u : v for which v < b + d.

With these preparatory results in hand, we can turn to the process of interpolating ratios between the two initial ratios 1 : 1 and 2 : 1. We need to realise that 1 : 1 is in close sequence with

[39] One could also call such pairs Farey pairs, or Farey neighbours.

[40] Note that as a result of this definition if a : b is in close sequence with c : d then a : b < c : d, and accordingly the relation of "being in close sequence" is not symmetrical.

[41] Between two ratios a : b and c : d, the interpolated ratio (a + c) : (b + d) is called their *mediant*.

2 : 1. Hence the mediants interpolated between neighbouring ratios a : b and c : d, will also be in close sequence with their neighbours at every step of interpolation. Then in order to see that the resulting sequence will contain every ratio p : q, we should proceed in a way so that we interpolate at step *n* all the mediants with second member *n*, resulting from the operation (a + c) : (b + d) on two neighbouring ratios in close sequence with each other:

[1]	1:1										2:1
[2]	1:1					3:2					2:1
[3]	1:1			4:3		3:2		5:3			2:1
[4]	1:1		5:4	4:3		3:2		5:3	7:4		2:1
[5]	1:1	6:5	5:4	4:3	7:5	3:2	8:5	5:3	7:4	9:5	2:1

Table 3

Now it is clear that with the completion of step *n* the sequence contains ratios where neighbouring ratios are in close sequence with each other, and the sum of the second members of neighbouring ratios is greater than *n*. One can show that in this sequence of ratios each and every ratio between 1 : 1 and 2 : 1 expressible with a second member equal to, or smaller than *n* is present. For suppose any one might be missing (in the sense that there is no ratio proportional to it on the list). In this case, the missing ratio has to be inserted somewhere between two ratios being in close sequence with each other. As said, the sum of the second members of these ratios is greater than *n*. But then, by our previous considerations, we cannot find a ratio between them with a second member smaller than the sum of the second members of these two ratios. Hence, the ratio we would have wanted to insert somewhere has already to be present on the list. By filling in the ratios in the specified order then, we shall interpolate the ratio p : q, where p and q are relative primes, exactly on step q of our procedure.*

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* The claim that a procedure of interpolating mediants between Farey neighbours is a problematic heuristic device for assigning the ratio of longest day and shortest night to particular locations (together with a previous version of the Appendix) I formulated some time ago in a manuscript. I am grateful to Alan Bowen for detailed comments and criticism of that manuscript. This paper – apart from some paragraphs of that central claim – was written in 2017, during a sabbatical leave from Eötvös Loránd University, and was also supported by the project OTKA K-112253.

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